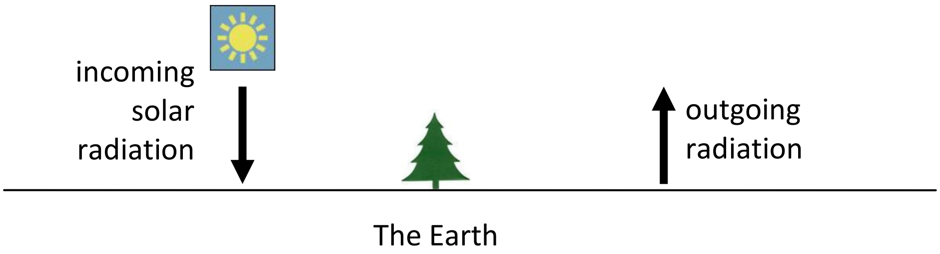
In this activity you will create spreadsheets to model what would happen to the temperature of the Earth if there were to be a sudden change in the amount of radiation entering or leaving our planet.

You will then investigate which functions best model the results you obtain.

Information sheet

At the most fundamental level, the temperature of the Earth is governed by the difference between the amount of **energy** the Earth receives from the Sun, and the amount of energy the Earth loses to Space.



The **outgoing radiation** depends only on the temperature of the Earth, T kelvins (K).

The amount of outgoing radiation is given by the Stefan-Boltzman Law:

Outgoing radiation = σT^4 where **σ = 5.67 $\times$ 10^{-8}** $\text{Js}^{-1}\text{m}^{-2}\text{K}^{-4}$

Assume that the (average) temperature of the Earth's surface is 283 K.

Then the outgoing radiation = σT^4 = 364 $\text{Js}^{-1}\text{m}^{-2}$ (to 3sf).

This is the energy lost per square metre of the Earth's surface per second.

The **temperature** of the Earth depends on the difference between the incoming and outgoing radiation and the heat capacity of the Earth.

If the incoming radiation is equal to the outgoing radiation (in this case 364 $\text{Js}^{-1}\text{m}^{-2}$), the temperature of the Earth is constant. If the incoming radiation and outgoing radiation are different, then the temperature of the Earth will change.

The following formulae can be applied over successive time increments to predict what will happen to the temperature as time passes.

Change in temperature of the Earth

$$= \frac{(\text{incoming radiation} - \text{outgoing radiation}) \times \text{time in seconds}}{\text{heat capacity}}$$

where the heat capacity of the Earth = **4×10^8** $JK^{-1}m^{-2}$

New temperature of the Earth = old temperature + temperature change

Over its lifetime the Sun's luminosity, the amount of energy it emits each second, has increased and it will continue to do so. Suppose that the Sun suddenly increased its luminosity by 5% and then stayed the same.
What would happen to the Earth's temperature? How long would this take?

Think about ...

- Check you can do this calculation: the outgoing radiation = σT^4 = 364 $Js^{-1}m^{-2}$ (to 3sf)
- What is the Kelvin temperature in Celsius?
- How do you enter numbers in standard form into a spreadsheet?
- In general terms, how would you expect the Earth's temperature to change after a sudden increase in radiation input?

Try these ...

In these activities you will use a spreadsheet to predict what will happen if the incoming radiation changes from the equilibrium value of 364 $Js^{-1}m^{-2}$ when the temperature of the Earth is 283 K. Use the information and formulae given above.

Open a new Excel spreadsheet.

1 First define the constants: put the value of σ in cell A1, the heat capacity of the Earth in cell B1, and a time increment in cell C1. Set this to half a year (making sure that this is in seconds to be consistent with the other units).

2 Put these table headings in cells A3 to E3:
Time, Incoming, Outgoing, Temperature change, Temperature

3 Specify the initial conditions:
In row 4 enter the temperature of the Earth at time 0 to be 283 K, and both the incoming and outgoing radiation to be 364 J $s^{-1}m^{-2}$.

Increase in incoming radiation

Use the spreadsheet to predict what would happen if the incoming radiation increased by 5% and then remained constant. Follow the steps below.

4 First consider the time (column A).

Write a formula in A5 to give the time in A4 (0) plus the time increment.

The time will be displayed in seconds – a big and not very meaningful number; change your formula (not the constant!) to display the time in years.

Copy the formula in column A down to row 20. This should give you a series of times at half-yearly intervals.

5 Now consider the incoming radiation (column B).

Write a formula in B5 to calculate the incoming energy after an increase of 5%. Complete column B down to row 20, keeping the incoming radiation constant at this new value.

6 Next consider the outgoing radiation at time = 0.5 years (cell C5). Construct a formula in cell C5 to give the outgoing radiation based on the constant in cell A1 and the temperature of the Earth at time 0 in cell C4.

7 Now consider the temperature change at time = 0.5 years (cell D5). In D5, write a formula to calculate the temperature change:
= (incoming radiation – outgoing radiation) x time increment / heat capacity

8 Finally consider the temperature at time = 0.5 years (cell E5). In E5 write a formula to calculate the new temperature:

= old temperature + temperature change

9 Extend the series in columns C, D and E down to row 20 to see how temperature evolves in your model.

10 Draw a graph of temperature against time for the Earth. Describe how the temperature changes with time.

Finding functions to model the way the temperature rises

11 Make 4 copies of your graph.

12 Add a trendline, as follows, to the first 3 copies.
In each case set the intercept to 283 and display the equation.
1st copy – add a quadratic regression line (polynomial order 2)
2nd copy – add a cubic regression line (polynomial order 3)
3rd copy – add a quartic regression line (polynomial order 4).

13 Now try an exponential function.
Add another column to the spreadsheet to calculate values of the function $y = 283 + 3.53(1 - e^{-0.475t})$ where t is time from column A.
Add the graph of this function to the 4th copy of your graph.

Evaluating the functions as models of the temperature

14 Consider each of the functions you have found (quadratic, cubic and quartic), and the exponential function as models of this situation. For each model answer the following questions.

a How well does the model fit the temperature data?

b Do you think the model gives realistic predictions for later times?
(Note that you can quickly find out what the model predicts for later times, by extending the columns further than 20 rows and then altering your graphs to include these new values.)

Decrease in incoming radiation

Now use the spreadsheet to predict what would happen if the incoming radiation decreased by 5% and then remained constant. Follow these steps.

15 Make a copy of the previous worksheet.

16 Consider the incoming radiation (column B).

Change the formula in B5 so that it calculates the incoming energy after a decrease of 5%.

You should find that the other values in your spreadsheet change automatically. If you set the scale on the y axis earlier, you may need to change it now to show what is predicted to happen to the temperature.

17 Describe what the graph of temperature against time shows now.

Finding functions to model the way the temperature falls

18 The new trendlines and their equations should be given on the first three graphs. If not, redraw them.

19 Now alter the exponential function.
Change the formula in the last column of the spreadsheet so that it now calculates values of the function $y = 279.45 + 3.55e^{-0.45t}$ where t is time from column A.

Again Excel will update the graph, but you may need to alter the scale.

Evaluating the functions as models of the temperature

Consider each of the functions you have found (quadratic, cubic and quartic) and the exponential function as models of this situation.

For each model answer the following questions.

20 How well does the model fit the temperature data?

21 Do you think that the model gives realistic predictions for later times?

Extension

If you have time, find models for other percentage increases and decreases.

You could also try using different time increments.

You could investigate an ongoing small percentage increase in radiation

Reflect on your work

Why does an exponential function give a better long term prediction than any of the polynomials?